Electron-Electron Interactions in Graphene:
Current Status and Perspectives

June 9, 2015
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2 Is that a Kondo Effect?
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Background

The study of electron-electron interaction goes for a long history since the investigation of Drude Model, discussed by Paul Drude[3] and Arnold Sommerfeld[4].¹

¹which, shamelessly, claims that electrons propagate freely in a non-relativistic (Galilean invariant) way (Drude’s contribution), and under Fermi-Dirac statistics.
Introduction

The study of electron-electron interaction goes for a long history since the investigation of Drude Model, discussed by Paul Drude[3] and Arnold Sommerfeld[4].¹

The simplest system that actually considers the interaction and by the same time yields fabulous result is Mott Insulator system. By Hubbard Model, which mainly serves to simplify the onsite Coulomb interaction(repulsion) by a easily acceptable way

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\[
H = U(\hat{n}_{1\uparrow}\hat{n}_{1\downarrow} + \hat{n}_{2\uparrow}\hat{n}_{2\downarrow} + \cdots)
\]

\(^1\)which, shamelessly, claims that electrons propagate freely in a non-relativistic (Galilean invariant) way (Drude’s contribution), and under Fermi-Dirac statistics.
With respect to time, the field theory has been applied to this problem.\(^2\) Hence, collective modes of electron such as screening effect has been studied well.

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Besides, 1D electron system’s boson excitation Luttinger Liquid and 0D electron system Quantum Dot are also well-studied, but we don’t talk them today.

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Besides, 1D electron system’s’s boson excitation Luttinger Liquid and 0D electron system Quantum Dot are also well-studied, but we don’t talk them today.

Another interesting phenomenon, which we previously reviewed, is Kondo effect involving magnetic impurities as an additional weird electron resistance at low temperature region.

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Last class, we talk about Klein paradox in graphene as an unexpected barrier to construct localized state. Vacancies defect, however, contributes to generate such a state, but meanwhile cause local magnetic state.

The heuristic criterion that describes the formation of a local magnetic moment is addressed at the mean field level by the Anderson impurity model, the system is described by Hamiltonian in momentum space.

\[ \mathcal{H}_V = V \sum_{\mathbf{p},\sigma} (f_{\sigma}^{\dagger} b_{\mathbf{p},\sigma} + b_{\mathbf{p},\sigma}^{\dagger} f_{\sigma}) \]
**Figure:** (a) Honeycomb lattice with an impurity atom. Black: sublattice A; White: sublattice B. Intersection of the Dirac cone spectrum, $E(k) = \pm v|k|$, with the localized level $E_f = \varepsilon_0$: (b) $\varepsilon_0 > 0$, (c) $\varepsilon_0 < 0$. 

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**E-E Interaction in Graphene**

**Introduction**

a) 

b) c)
Hence, we begin to ask how electron-impurity interaction results in some visible effect. However, the typical effect, Kondo effect, the coupling of conduction electrons to local magnetic moments which we have already known a lot and at the same time is a central problem in CMP, has not been realized in graphene for a long time.

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Technically, Kondo effect could not be considered as E-E interaction. But interestingly, the magnetic phenomenon is only possible to detect by transport experiment and the STM is out due to the scale of difference in DOS.

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Trick goes here. It’s hard to distinguish the difference between different causes in transporting property. We are going to talk this particular idea in detail.

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Tunable Kondo effect in graphene with defects

Jian-Hao Chen\textsuperscript{1,2†}, Liang Li\textsuperscript{2}, William G. Cullen\textsuperscript{1,2}, Ellen D. Williams\textsuperscript{1,2} and Michael S. Fuhrer\textsuperscript{1,2*}

\textbf{Figure}: Main Experiment Paper Discussed Today\cite{2}
Tunable Kondo effect in graphene with defects

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Maybe I can ask a question here: What’s the most important property of Kondo effect?
Tunable Kondo effect in graphene with defects

Jian-Hao Chen¹²⁺, Liang Li², William G. Cullen¹², Ellen D. Williams¹² and Michael S. Fuhrer¹²★

Figure: Main Experiment Paper Discussed Today[2]

Maybe I can ask a question here: What’s the most important property of Kondo effect?

Definitely the logarithms resistivity at low temperature.
Figure: Universal Kondo behaviour of graphene with defects.

**a**, Temperature-dependent resistivity $\rho(V_g)$ of graphene sample Q6 under a perpendicular magnetic field of 1 T, at 12 different gate voltages, with temperature changing from 300 mK to 290 K.

**b**, The normalized Kondo part of the resistivity $(\rho - \rho_{c1})/\rho_{K,0}$ versus $T/T_K(V_g)$, where $T_K(V_g)$ is the Kondo temperature at respective gate voltage. The red line is the expected universal Kondo behaviour from numerical renormalization group calculations.
We have already done with this paper. What’s next?

It is this paper that causes widely academical debates, for mainly:

- Kondo effect is extremely difficult to achieve in graphene.
- The seemingly major phenomenon can be derived through few other ways.
  a) Electron-Electron Interaction
  b) Weak Localization
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Is that a Kondo Effect?
Why Kondo Unaccessible?

General explanation to the accessibility

For a detailed review, see [5].
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- In metal, the magnetic screening (the ultimate consequence of the so-called Kondo cloud) causes suppression the appearance of magnetic moments.
- However, in graphene, the effect is suppressed mainly due to
  a) Low density of states (chemical potential)
  b) Sublattice structure (Symmetry breaking or not)
Some theoretical picture

From a tight-binding perspective, for a spin 1/2 impurity, the hybridization Hamiltonian can be written in the diagonal basis:

$$\mathcal{H} = V \sum_{\alpha=\pm} \sum_{p,\sigma} \left[ \Theta_{\alpha,p} c_{\alpha,p}^\dagger f_{\sigma} + h.c. \right]$$

where

$$c_{\pm,k\sigma} = \frac{1}{\sqrt{2}} \left[ b_{k\sigma} \pm \left( \frac{\phi_k^*}{|\phi_k|} \right) a_{k\sigma} \right]$$
As in metals, the Anderson Hamiltonian in graphene can be mapped into the spin exchange Hamiltonian by a canonical transformation.
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The spin exchange Hamiltonian between the magnetic adatom and the graphene electrons is

$$\mathcal{H}_e = -J \sum_{kk'} \sum_{\alpha \alpha'} \Theta^*_{\alpha, \mathbf{k}} \Theta_{\alpha', \mathbf{k}'} \mathbf{S} \cdot \mathbf{c}_{\alpha', \sigma', \mathbf{k}'} \sigma \mathbf{c}_{\alpha, \sigma, \mathbf{k}}$$

where $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices, $\mathbf{S} = \frac{1}{2} f_\sigma^\dagger \sigma f_{\sigma'}$ is the localized spin.
E-E Interaction in Graphene

Is that a Kondo Effect?

Why Kondo Unaccessible?

Figure: Unlike the situation in metals, the exchange coupling in graphene can be controlled by gating, in particular when the chemical potential is brought to the proximity of the localized level, where the Kondo coupling becomes resonant.
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The critics start from Weber et al., who comment that the resistivity data might be confounded by Altshuler–Aronov corrections, which comes from electron-electron interaction. It also contributes a logarithmic rise of resistivity at low temperatures in 2D system[1]:

$$\Delta \rho \propto \frac{1}{\pi} \frac{\nu(q)}{\epsilon_F} \ln(|\epsilon|\tau)$$

\(^4\)In a 3D system, however, it has a form of \((\epsilon\tau)^{1/2}(\epsilon_F\tau)^{-2}\)
The critics start from Weber et al., who comment that the resistivity data might be confounded by Altshuler – Aronov corrections, which comes from electron-electron interaction. It also contributes a logarithmic rise of resistivity at low temperatures in 2D\(^4\) system\(^1\):

\[
\Delta \rho \propto \frac{1}{\pi} \frac{v(q)}{\epsilon_F} \ln(|\epsilon|\tau)
\]

Others also comment on the probability of weak localization.

\(^{4}\)In a 3D system, however, it has a form of \((\epsilon\tau)^{1/2}(\epsilon_F\tau)^{-2}\).
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To apply in a graphene system, the resistivity can be expressed by

$$\rho_{xx} = \rho_{xx,0} \left[ 1 + A \frac{\rho_{xx,0} e^2}{2\pi^2 \hbar} \left( \mu^2 B^2 - 1 \right) \ln \left( \frac{k_B T \tau_{tr}}{\hbar} \right) \right]$$

where $\rho_{xx,0}$ is the uncorrected longitudinal resistivity and $A$ is a constant less than $1^5$, $\mu$ the charge carrier mobility, $\tau_{tr}$ the relaxation time of transport momentum.

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$^5$If $A > 1$, the system is unphysical. In similar cases, the range of $A$ is experimental fitted by $A \sim 0.3 - 0.9$. 
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\]

where \(\rho_{xx,0}\) is the uncorrected longitudinal resistivity and \(A\) is a constant less than \(\frac{1}{5}\), \(\mu\) the charge carrier mobility, \(\tau_{tr}\) the relaxation time of transport momentum.

It has two parameters, \(T\) and \(B\). We can check the fitting of the two parameters.

\[^{5}\text{If } A > 1, \text{ the system is unphysical. In similar cases, the range of } A \text{ is experimental fitted by } A \sim 0.3 - 0.9.\]
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Is that a Kondo Effect?

Rule out other cases

Figure: Temperature and magnetic-field dependent resistivity of graphene with defects. **a**, Magnetoresistance of graphene sample Q6 with defects at a temperature $T = 300 \text{mK}$ and at gate voltages $V_g - V_{g,\text{min}} = 15.3 \text{V}$ (blue stars), $-25.3 \text{V}$ (purple triangles), $-35.3 \text{V}$ (red circles) and $-45.3 \text{V}$ (green crosses); $V_{g,\text{min}} = 5.3 \text{V}$ is the gate voltage of minimum conductivity. **b**, Temperature-dependent resistivity of graphene under $1T$ of transverse magnetic field and at the same four $V_g$ values in **a**. In **a** and **b** the solid lines are with $A = 0.32$, chosen to fit the data in **a**.
It can be seen that the correction is plausible only when temperature is very low; it doesn’t account the saturation Temperature effect.

\[ L_T > l_{sample}, \text{ and an approximation of } L_T \text{ is } L_T \approx \frac{\mu}{v_F e} \sqrt{\frac{E_F^3}{k_B T}} \]
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The saturation and related temperature can be explained for a revised version of A-A effect, which take the finite size of the sample into account; however, this sort of modification only relates to the gate voltage $|V_g - V_{g,\text{min}}|$ by

$$k_B T_{\text{sat}} \approx \frac{\mu^2 E_F^3}{v_F^2 e^2 l_{\text{sample}}^2}$$

where, the gate voltage influences this by $|V_g - V_{g,\text{min}}| \propto E_F^2$. The $l_{\text{sample}}$ is nothing but the shortest sample dimension.\(^6\)

\(^6\)This expression is given by consider the Temperature-dependent thouless length $L_T > l_{\text{sample}}$, and an approximation of $L_T$ is $L_T \approx \frac{\mu}{v_F e} \sqrt{\frac{E_F^3}{k_B T}}$.
Based on this, we can go on to make theoretical assumption of the approximately saturation temperature from the following figure:
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Figure: This figure illustrates the theoretical result (solid lines) and experimental results (dashed point) of how the saturation temperature changes with $|V_g - V_{g,min}|$. A-A effect predicts a strong $\Delta V_g$ dependency, and the two different-size samples should separated, while the experimental data proves neither.
These are all bad results. If you’re still unsatisfied of the result, what if I claim the fitting gives $A \sim 1.9 - 2.5$ in $b$ in the previous figure?
Is that weak localization?

Remember what is a weak localization?
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*Weak localization is a phenomenon caused by the quantum correction to the conductivity of two dimensional systems due to electron interference.*

– *PhysRevLett.100.056802[6]*
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Weak localization is a phenomenon caused by the quantum correction to the conductivity of two dimensional systems due to electron interference.

– PhysRevLett.100.056802[6]

What is its correction in graphene? As you might guess, it’s logarithmically.
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Is that a Kondo Effect?

Rule out other cases

Figure: A simple illustration
You might say ‘yooooo’, but it’s not applicable in our experiment.
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Because such a correction by WL would be completely destructed due to slightly magnetic field applying on it, while in our experiment, the $B$-dependency is not this form.
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Raman illustration of defects

Figure: After irradiation and annealing overnight at 490K in UHV, the $D$ peak which stands for defects, arisen to very high. Recall what $D$ peak is? (compare to $G$, i.e., $2D$ peak)
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3. Reference
B. L. Altshuler, A. G. Aronov, and P. A. Lee.
Interaction effects in disordered fermi systems in two dimensions.

Jian-Hao Chen, Liang Li, William G Cullen, Ellen D Williams, and Michael S Fuhrer.
Tunable kondo effect in graphene with defects.

Paul Drude.
Zur elektronentheorie der metalle; ii. teil. galvanomagnetische und thermomagnetische effecte.
